Erratum for *Regularized Stokeslet Surfaces*

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This note corrects some typographical errors in the paper [Ferranti, Cortez, *Regularized Stokeslet Surfaces*, *Journal of Computational Physics*: 508:113004]. These corrections have no bearing on the numerical results in the manuscript, which came from code that implemented the method correctly. For those interested, Matlab code written by the author of this note has been made available at

`https://github.com/djferranti/RegularizedStokesletSurfaces`.

The authors of the original paper apologize for any inconvenience these errors may have caused.

1. Formula (2.22) was missing a factor of $\binom{m+n}{k}$ in the sum and should read

$$A_{m,n,q} = S^{\widehat{\mathbf{e}}_2}_{n,q} - \sum_{k=0}^{m+n} \binom{m+n}{k} (-1)^k S^{\widehat{\mathbf{d}}}_{k,q}$$

   where here and in the rest of this document, we use red to highlight the correction. The correction here should also be made to the last line of the derivation in the Appendix A.1 .

2. Near the top of section 2.2.3, we should have:

   We note that the surface Laplacian of $R(r)$ is

$$\Delta_{2D}(R) = \frac{1}{R} + \frac{\gamma^2}{R^3}$$

   Here, γ^2 replaces the ϵ^2.

3. In Section 2.2, there is a correction because of a mix-up between gradients in physical space versus gradients in parameter space. Since we are evaluating a line integral in physical space, the gradients should all be in physical space. For example, take our definition of

$$r^2 = r^2(\alpha, \beta) = (\mathbf{x}_0 \cdot \widehat{\mathbf{v}} + \alpha L)^2 + (\mathbf{x}_0 \cdot \widehat{\mathbf{n}} + \beta H)^2$$

   Here, α, β are nondimensional parameters. To take the gradient of r in *physical* space, we let $s_1 = L\alpha$ and $s_2 = H\beta$. Then,

$$r^2 = r^2(s_1, s_2) = (\mathbf{x}_0 \cdot \widehat{\mathbf{v}} + s_1)^2 + (\mathbf{x}_0 \cdot \widehat{\mathbf{n}} + s_2)^2$$

and

$$\nabla r = \nabla_{s_1,s_2} r = \frac{1}{r}\begin{bmatrix} \mathbf{x}_0 \cdot \widehat{\mathbf{v}} + s_1 \\ \mathbf{x}_0 \cdot \widehat{\mathbf{n}} + s_2 \end{bmatrix}$$

where we have emphasized the gradient taken is in physical space by the notation ∇_{s_1,s_2}.

If we return to nondimensional variables, we get the following corrections for ∇r:

$$\nabla r = \frac{1}{r}\begin{bmatrix} \mathbf{x}_0 \cdot \widehat{\mathbf{v}} + L\alpha \\ \mathbf{x}_0 \cdot \widehat{\mathbf{n}} + H\beta \end{bmatrix}$$

Comparing this to the expression for ∇r on page 7, we notice there are erroneous factors of L, H in the first and second entries of the vector respectively. The same issue appears in ∇R and $\nabla \psi$.

Unfortunately, this error affects some of the other expressions in Section 2.2. For formula (2.28),

$$\begin{aligned} &\int_{S_1} \nabla \psi \cdot \widehat{\mathbf{n}}\, ds \\ &= -\frac{(\mathbf{x}_0 \cdot \widehat{\mathbf{n}})}{\gamma L} \int_0^1 \frac{1}{\sqrt{(\alpha + P)^2 + Q^2}\left(\sqrt{(\alpha + P)^2 + Q^2} + \gamma / L\right)}\, d\alpha \end{aligned}$$

where additionally a factor of L was missing in the original manuscript ($ds = L\, d\alpha$).

In Algorithm 1, we should have

$$\text{out}_i \leftarrow -\frac{(\mathbf{x}_0 \cdot \widehat{\mathbf{n}})}{\gamma L} \times (2.30)$$

and similarly

$$\text{out}_i \leftarrow -\frac{(\mathbf{x}_0 \cdot \widehat{\mathbf{n}})}{\gamma L} \times (2.29)$$

And in formula (2.32), we should have

$$\begin{aligned} \int_{S_1} \frac{\partial R}{\partial n}\, d\mathbf{s} &= -\frac{(\mathbf{x}_0 \cdot \widehat{\mathbf{n}})}{L} \int_0^1 \frac{1}{\sqrt{(\mathbf{x}_0 \cdot \widehat{\mathbf{v}} + \alpha L)^2 + (\mathbf{x}_0 \cdot \widehat{\mathbf{n}})^2 + \gamma^2}}\, d\alpha \\ &= -\frac{(\mathbf{x}_0 \cdot \widehat{\mathbf{n}})}{L} \operatorname{arctanh}\left(\frac{\mathbf{x}(\alpha) \cdot \widehat{\mathbf{v}}}{R(\mathbf{x}_f, \mathbf{y}(\theta))}\right)\Bigg|_{\alpha=0}^{\alpha=1} \end{aligned}$$

One final note: At the end of Algorithms 1, we should have $T_{0,0,3} \leftarrow T_{0,0,3}/(LH)$ before returning $T_{0,0,3}$. This is in order to be consistent with formulas (2.9, 2.10). A similar statement applies to $T_{0,0,1}$ in Algorithm 2.